

Signs of quadratic function

Part 4



Parametric equation

A parametric equation is an equation that is depending on a parameter m .

Example:

- ❖ $mx + 4 = 0$ is a parametric equation of first degree of main variable x and parameter m .
- ❖ $(t + 1)x^2 + 2x - 3t = 0$ is a parametric equation of main variable x and of parameter t .

In this video, we will learn how to solve and discuss the existence of roots of a parametric equation in form of $ax^2 + bx + c = 0$
(as in the second example)



Parametric example #1

Consider the parametric equation:

$$(m + 1)x^2 + 2x + m = 0 ; m \in \mathbb{R}$$

The coefficients are: $a = m + 1$; $b = 2$; $c = m$

Case 1: solve the equation for $a = 0$

- $a = 0$; $m + 1 = 0$; $m = -1$
- The equation becomes: $2x - 1 = 0$
- The solution is: $x = \frac{1}{2}$ single (unique) solution



Parametric example

Consider the parametric equation:

$$(m + 1)x^2 + 2x + m = 0 ; m \in \mathbb{R}$$

Case 2: solve the equation for $a \neq 0$

Step 1: Calculate the discriminant Δ (or Δ')

$$a \neq 0 ; m \neq -1$$

$$\Delta' = b'^2 - ac = 1 - m(m + 1) = 1 - m^2 - m = -m^2 - m + 1$$



Parametric example

Consider the parametric equation:

$$(m + 1)x^2 + 2x + m = 0 ; m \in \mathbb{R}$$

Step 2: Discuss according to the values of m the signs of the discriminant

$$\Delta' = -m^2 - m + 1 \quad \text{Quadratic function}$$

$$\delta_m = b^2 - 4ac = (-1)^2 - 4(-1)(1) = 1 + 4 = 5 > 0$$

$$m_1 = \frac{1-\sqrt{5}}{-2} = \frac{-1+\sqrt{5}}{2} ; \quad m_2 = \frac{1+\sqrt{5}}{-2} = \frac{-1-\sqrt{5}}{2}$$

m	$-\infty$	m_2	-1	m_1	$+\infty$
Δ'	$-$	0	$+$	0	$-$
	S		O		S



Parametric example

Consider the parametric equation:

$$(m + 1)x^2 + 2x + m = 0 ; m \in \mathbb{R}$$

Step 3: Discuss the existence and the number of solutions of the equation

❖ If $m \in] - \infty ; m_2 [\cup] m_1 ; + \infty [$:

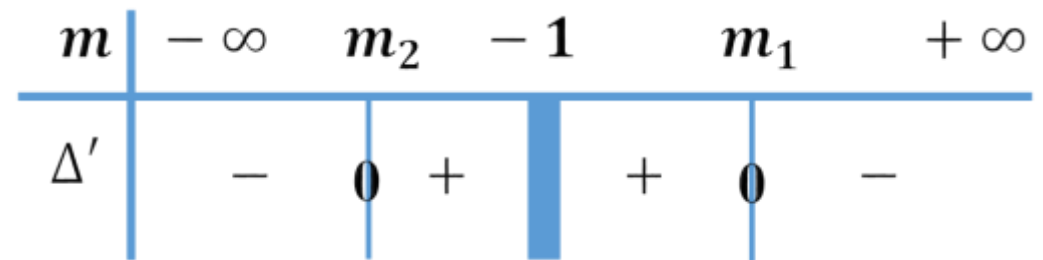
no real roots

❖ If $m \in] m_2 ; m_1 [$:

2 distinct roots $x = \frac{-b' \pm \sqrt{\Delta}}{a} = \frac{-1 \pm \sqrt{-m^2 - m + 1}}{m + 1}$

❖ If $m \in \{m_1 ; m_2\}$

1 double root $x_1 = x_2 = -\frac{b'}{a} = \frac{-2}{m+1} = \begin{cases} \frac{-4}{1+\sqrt{5}} & \text{if } m=m_1 \\ \frac{-4}{1-\sqrt{5}} & \text{if } m=m_2 \end{cases}$



Parametric example #2

Consider the parametric equation:

$$(1 + m^2)x^2 + 2\sqrt{3}x - 1 = 0 ; m \in \mathbb{R}$$

The coefficients are: $a = 1 + m^2$; $b = 2\sqrt{3}$; $c = -1$

$a = 1 + m^2 \neq 0$ for all values of m

$\Delta' = 3 - (-1)(1 + m^2) = 3 + 1 + m^2 = 4 + m^2 > 0$ for all values of m .

The equation has two distinct roots for all values of m :

$$x = \frac{-b' \pm \sqrt{\Delta'}}{a} = \frac{-\sqrt{3} \pm \sqrt{4 + m^2}}{1 + m^2}$$



Parametric example #3

Consider the parametric equation:

$$x^2 + 2x + m^2 + 1 = 0 ; m \in \mathbb{R}$$

The coefficients are: $a = 1$; $b = 2$; $c = m^2 + 1$

$a = 1 \neq 0$ for all values of m

$\Delta' = 1 - (1)(1 + m^2) = -m^2 = -m^2 \leq 0$ for all values of m .

❖ If $m = 0$; $\Delta' = 0$

The equation has one double root: $x_1 = x_2 = -\frac{b'}{a} = -\frac{1}{1} = -1$

❖ If $m \neq 0$; $\Delta' < 0$

The equation has no real roots



Now it is your turn
Can you solve this parametric equation????

$$x^2 + (3m - 4)x + 4 = 0; m \in \mathbb{R}$$



